AN ALIGNMENT CHART FOR DOUBLE THEODOLITE COMPUTATIONS

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When exceptionally precise measurements of wind velocities are needed, usually in connection with research problems, the double-theodolite pilot balloon method is favored. The disadvantage of extra personnel and more complicated calculations is outweighed by the advantage of a more certain result. The method uses these two formulas:

$$d=\frac{b \sin \beta}{\sin (\alpha\text{-}\beta)} \tag{1}$$

$$h=d \tan e \tag{2}$$

α and β are azimuths measured at the two observation points A and B; d is the horizontal distance of the balloon from station A; b is the length of the baseline (distance A to B); e is the elevation angle at station A; and h is the balloon's height. The formulation neglects curvature of the earth. These equations are usually evaluated on a slide-rule or a desk calculator. An alignment chart for solving them will be described. Compared with other methods it has some definite computational advantages, among them speed, simplicity, and economy.

Since both the equations are of the same general type,

$$f(y)\,f(z)-f(x)=0 \tag{3}$$

they may be solved by a three-variable alignment chart. The circular form has been chosen for a reason which will become clear. The basic equation for such a chart is

$$\begin{vmatrix} \frac{1}{1+[f(y)]^2} & \frac{f(y)}{1+[f(y)]^2} & 1 \\ \frac{1}{1+[f(z)]^2} & \frac{-f(z)}{1+[f(z)]^2} & 1 \\ \frac{1}{1+f(x)} & 0 & 1 \end{vmatrix}=0 \tag{4}$$

(The reader can easily verify that (4) reduces to (3) if $f(y)+f(z)\neq 0$.) This chart will have an x-scale along the diameter of, and y- and z-scales along the arcs of a circle. The left hand column of (4) gives the components along the diameter of $f(x), f(y)$, and $f(z)$, and the middle column gives components perpendicular to the diameter. For example, the scale for h and d is laid out along the diameter from the left (taken as the origin), distances being calculated by

$$\frac{1}{1+f(x)}=\frac{1}{1+d}, \text{ or}=\frac{1}{1+h}. \tag{5}$$

Figure 1 shows one form of the completed chart in the special case of a unit baseline, $b=1$. The construction and labeling of the chart may be clearer if the reader will substitute a few values of the variables, such as $d=0$, $d=1$, $\beta=0°$, $\beta=90°$, $e=45°$, and so on, into the proper terms of (4) and locate these points for himself.

The chart is extremely simple to use. To find d and h;

(a) lay a straight edge (or stretch a string) from observed β to observed $(\alpha-\beta)$ and read d where this crosses the diameter.

(b) lay a straight edge from d (this time the scale on the circle) to e and read h where this crosses the diameter.

The dashed lines illustrate a sample problem where $\beta=30°$ $(\alpha-\beta)=10°$, and $e=40°$. The chart assumes a unit baseline, and so the d and h scales coincide. The values of d and h must be multiplied by the actual baseline. In this problem, if the actual baseline were 1000 yds., d would equal 2900, and h would equal 2300 yds. In an actual application, this multiplication would be performed in advance in laying out the d-scale and separate h- and d-scales would be provided.

A chart the size of figure 1 is only useful for the purpose of illustration. It can be entered to the nearest degree or so, and might be used for rough calculations. However, a plotting board with a rotatable plastic disc about a yard in diameter is now in use, for the final part of the upper wind calculation, to determine the wind vector. An enlarged version of figure 1 was printed on photographic film and attached to the back of one of these discs. The scale layout was simplified by rotating the $(\alpha-\beta)$-scale 180°, and the d-scale 90°, clockwise about the circle. The straightedge was fastened by a bracket so as to pivot directly above the circle at point A, figure 2. Arrows B and C are permanent marks on the plotting board, located exactly 90° and 180° clockwise around the circle from A. On a chart this size, most angles can be located to the nearest tenth of a degree or less, and values of d and h

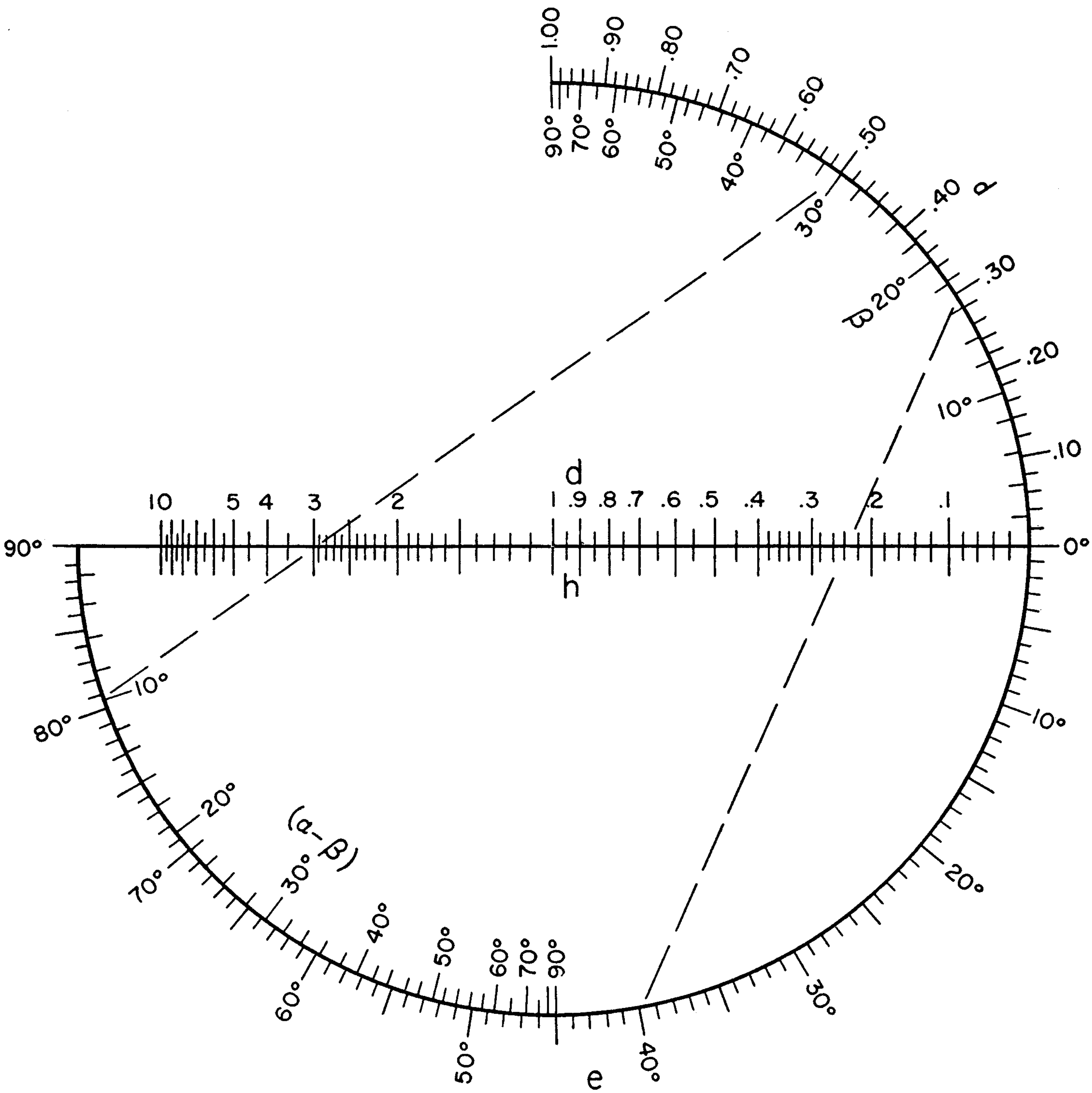

FIGURE 1.—Alignment chart for graphical solution of double-theodolite equations (1) and (2). Dashed lines refer to sample calculations in text.

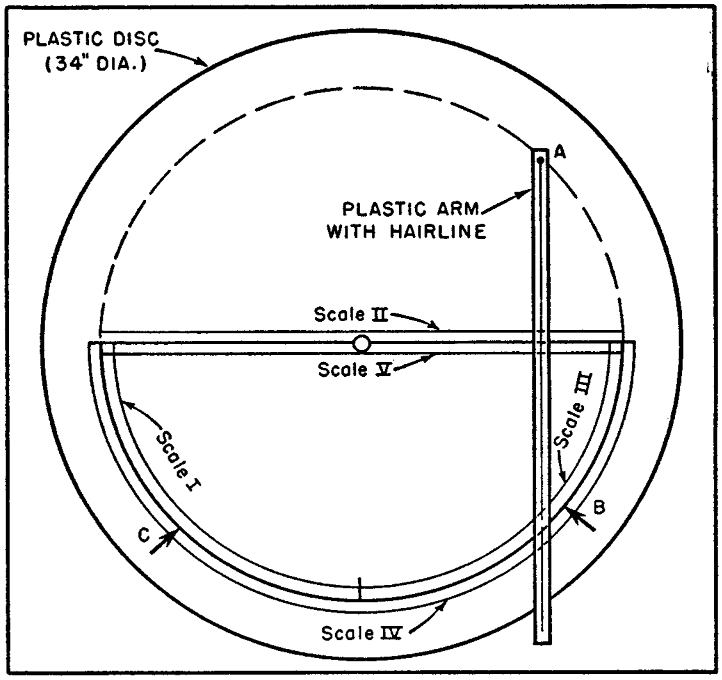

FIGURE 2.—Schematic drawing of enlarged version of figure 1 attached to plastic protracter of upper winds plotting board, showing location of movable plastic arm, pivoted at A, and of arrows at B and C, fixed to the base.

read to within about 2 percent. To find d and h on such a device:

(a) set β, scale I, to arrow C; set the straightedge to $(\alpha-\beta)$ on scale I, and read d on scale II.

(b) set d on scale III to arrow B; set the straightedge to e on scale IV and read h on scale V.

From preliminary tests, it appears that, with the help of this device, double-theodolite runs can be made and simultaneously worked up by a team of three observers, the recorder (third man) performing all calculations on the plotting board during the progress of the run. This means that with little extra effort, the results of a double-theodolite run can now be made available shortly after the run ends. This will certainly be valuable in research applications, but will be even more so if routine double theodolite observations on an operational basis are encouraged.

The need for a faster and more convenient method of double-theodolite calculation was pointed out to the writer by Mr. DeVer Colson, U. S. Weather Bureau, whose interest and valuable assistance are gratefully acknowledged.